The Tuning Script from Bach's Well Tempered Clavier: A Possible 1/18th PC Interpretation

(Revised June 2nd, 2006, after original essay of 3-3-05)

by Daniel Jencka

Bradley Lehman has brought the tuning of J.S. Bach's *Well Tempered Clavier* back under the spotlight. The present inquiry was inspired by his writings (http://www.larips.com) on the series of loops conspicuously penned atop the title of Bach's "fair copy" WTC manuscript, recently decoded by Lehman and written up in his article for the February and April 2005 issues of *Early Music*. My reinterpretation of this possible WTC tuning script (for ease of comparison viewed upside down as Lehman does, with his placement of tuning series notes as in *Figure 1*) yields a slightly different temperament, affecting G#, D# and A# by slightly raising their pitches relative to Lehman's 1/12th PC interpretation. My 1/18thPC approach also has a pure rather than a wide 5th from A#/Bb to F. It also yields only three sizes of 5ths rather than four.

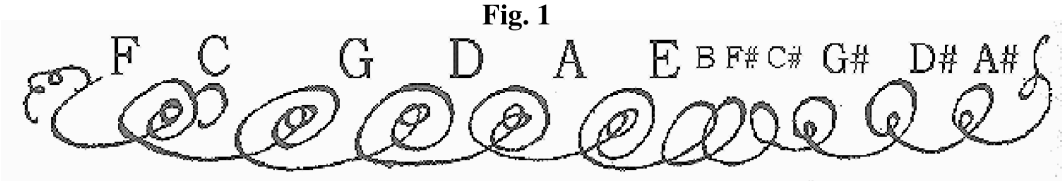
A couple of things drew me to explore Lehman's "Bach-Lehman" 1/12th PC interpretation (hereafter referred to as the "B-L1/12thPC") and to think about the WTC script's genesis, language, and interpretation. To be honest, I had a slight prejudice against wide 5ths appearing in a Baroque circulating temperament, no doubt from my long ago readings of piano tuner Owen Jorgensen's books, but also because the historical "well" or circulating temperaments I've used over the years had only narrow or pure 5ths in them. I knew that some French temperaments "ordinaire" (and, I've since learned from Bradley and others on the harpsichord list, a few German circulating temperaments of Werkmeister as well) include some distinctly wide 5ths. Nonetheless, this A#/Bb to F wide 5th was mildly perturbing to me. For the best reasons or not, it drew my attention.

I also thought that the Bb major 3rd sounded wide in the B-L1/12thPC, relative to major 3rds on F and Eb, such that the F-A, Bb-D and Eb-G major 3rds do not progressively widen in an even manner, as one might anticipate in a sophisticated "well" temperament (*perhaps* used by J.S. Bach). At least to my ear, the Bb in the B-L1/12thPC sounded slightly low as a minor 6th over D, and in harmonies where it is a 7th or some other dissonance. I found the sound of the slightly different tuning resulting from the proposed 1/18th PC temperament to be slightly more agreeable in the flat keys, and in it's gradually widening 3rds as one moves away from C major into both flat and sharp keys.

It all began. . .

One uncommonly wet and stormy night (for San Diego) I was staring at that single small loop conspicuously floating off the tail end of the WTC script. *(Figure 1)*

Fig. 1



Was it merely decorative? Just a little flourish to end the single long stroke that created the script? Or does it mean something quite definite? If so, then what meaning could it impart? In Lehman's approach, barrowed from Andreas Sparschuh's conceptual overlay of 1999 (http://www.strukturbildung.de/Andreas.Sparschuh/) where the eleven looping figures of the script were taken to represent a series of 5ths used in tuning, multi-looped figures standing for tempered 5ths and the three single loop figures signifying pure 5ths. My inquiry began with the conjecture that the somewhat curious single loop floating off at the end of the script also signified a (final) pure 5th, specifically from Bb to F. I of course wondered why it would have been drawn smaller than the three pure 5th loops in the body of the script, almost like an afterthought. But such questions would only be worth exploring if a pure 5th in that location resulted in a workable temperament that made sense of the entire tuning script.

So the first step in this investigation was to see how incorporating a pure A#/Bb to F 5th would shape the temperament, and then how that compared to the B-L1/12thPC. I soon saw that if this 5th was to be pure then the three preceding 5ths, C# to G# to D# to A#, would have to absorb the 1/6th comma into three similarly narrow 5ths, rather than two narrow and one wide, so therefore into three -1/18th comma 5ths. But what historical temperament ever used 18ths of a comma? None that I'd heard of. So I engaged in a thought experiment to see if and how this could possibly be what the old script's creator intended.

A simple approach to the tuning

How would Bach (or whoever may have devised such a temperament) come up with dividing the comma into an unheard of eighteen parts? Such a temperament's creator may have taken a common modified meantone approach, beginning with a 1/6th comma set of consecutive 5ths on the white keys, then by analysis and/or tinkering found that if one of those sixths of a comma were redistributed over say three 5ths, the result could end up being a fine, "well" temperament. (Modified meantone temperaments are created by tuning some set of four or more regular 5ths on the naturals, and then one goes about adjusting 3rds to make for some agreeable tuning in most keys.) So a five -1/6ths and three -1/18ths PC temperament could have come about through trial and error tweaking, from an abstract methodical analysis, or by some combination of the two.

If our imaginary temperamentician began with F-C-G-D-A-E-B as –1/6th tempered 5ths, a good potential 1/6th comma to redistribute would be that of E-B, thereby leaving a sequential core of regular 5ths on tones sensibly relating to open strings on violin family instruments, as Lehman has pointed out. (Bach was himself a string player.) The next task would be to locate the accidentals, and also perhaps think about what the overall shape of a good circulating temperament may be. An ideal circulating temperament would have gradually widening major 3rds as one moved away from C in either direction, and this typically led to a wide and wild major 3rd peak at the most remote keys, typically on C# or F.

Anyway, making that E-B 5th pure produces a fine G-B major 3rd, making the next 5th of B-F# pure yields a fine D-F# 3rd, and continuing with a pure F#-C# 5th produces a good A-C# 3rd, with a desired gradual increase in the size of all these 3rds as well. Going from F towards the flats by a pure 5th produces a decent (and gradually larger than F-A) Bb-D major 3rd. We are now left with G#/Ab and D#/Eb, the two tones typically fiddled with the most in a modified meantone, the imagined starting point for this circulating temperament.

All it takes then, is to make barely narrowed 5ths, between the already tuned C# and Bb, by setting G# and D# to make a slightly wavering C#-G#-D#-A#. Voila, that remaining 1/6th comma is distributed and a circulating temperament results. It is worth wondering whether Bach, or whoever may have devised such a temperament, would care how precisely that 1/6th comma was eaten up by three narrowed fifths on C#, G# and D#, though of course anyone would have known by simple division that 18ths were involved in an even distribution. Knowing just that much would later make a graphical representation, a tuning script, possible to devise and execute.

This is how the 1/18PC looks in fractions of a comma and in Temperament Units:
F –1/6 **C** –1/6 **G** –1/6 **D** –1/6 **A** –1/6 **E** 0 **B** 0 **F#** 0 **C#** –1/18 **G#** –1/18 **D#** -1/18 **A#** 0 **F**
F -120 **C** -120 **G** -120 **D** -120 **A** -120 **E** 0 **B** 0 **F#** 0 **C#** -40 **G#** -40 **D#** -40 **A#** 0 **F**

And the B-L1/12thPC:
F -1/6 **C** -1/6 **G** -1/6 **D** -1/6 **A** -1/6 **E** 0 **B** 0 **F#** 0 **C#** -1/12 **G#** -1/12 **D#** -1/12 **A#** +1/12 **F**
F -120 **C** -120 **G** -120 **D** -120 **A** -120 **E** 0 **B** 0 **F#** 0 **C#** -60 **G#** - 60 **D#** -60 **A#** +60 **F**

And a comparison of major 3rd beat rates that differ between the B-L1/12th and the 1/18PC:
A@ 415
B-L1/12thPC: (**e – g#**) = 8.82 (**b – d^1#**) = 11.90 (**f# – a#**) = 15.86 (**g# – c^1**) = 8.91 (**d# – g**) = 11.67 (**a# – d^1**) = 7.49
1/18thPC: (**e – g#**) = 9.11 (**b – d^1#**) = 12.78 (**f# – a#**) = 17.84 (**g# – c^1**) = 8.54 (**d# – g**) = 10.57 (**a# – d^1**) = 6.25
A@ 440
B-L1/12thPC: (**e – g#**) = 9.35 (**b – d^1#**) = 12.60 (**f# – a#**) = 16.80 (**g# – c^1**) = 9.44 (**d# – g**) = 12.37 (**a# – d^1**) = 7.94
1/18thPC: (**e – g#**) = 9.67 (**b – d^1#**) = 13.56 (**f# – a#**) = 18.94 (**g# – c^1**) = 9.05 (**d# – g**) = 11.20 (**a# – d^1**) = 6.62

And finally, a comparison of major 3rd sizes, over root tones, in SC percentages, so regardless of pitch:
B-L1/12thPC:
Db 81.8%, **Ab** 72.7%, Eb 63.6%, **Bb** 54.5%, **F** 27.3%, **C** 27.3%, **G** 45.5%, **D** 63.6%, **A** 81.8%, **E** 90.9%, **B** 81.8%, **F#** 72.7%
1/18thPC:
Db 81.8%, **Ab** 69.7%, **Eb** 57.6%, **Bb** 45.5%, **F** 27.3%, **C** 27.3%, **G** 45.5%, **D** 63.6%, **A** 81.8%, **E** 93.9%, **B** 87.9%, F# 81.8%

(Note that I place "C" near the center of the series of 5ths because I think that is the most informative arrangement, corresponding to key signatures most commonly used in actual pieces from Bach's time, flat keys going off to the left up to five flats, and sharp keys off to the right up to six sharps.)

Comparing the two temperaments, one sees that major 3rds on E, B and F# are progressively wider, and those on Eb, Ab and Bb are progressively narrower, in the 1/18thPC compared to the B-L1/12thPC. Both temperaments peak with the widest major 3rd of E-G#, and while that peak interval is slightly wider in the 1/18thPC it is still well within norms for historic circulating temperaments. The most significant difference between the two interpretations lies in their overall shapes. The major 3rds in the B-L1/12thPC do not progress evenly away from C and back again, but rather rise to a peak of 90.9% on E, drop to 72.7% on F#, and then rise again to 81.8% on Db before dropping off to 27.3% on C and F. And, as noted near the beginning of this essay, there is a large doubling in size from F-A at 27.3% to Bb-D at 54.5%, such that the progression of major 3rd sizes from F to Bb to Eb is not at all gradual.

By comparison, major 3rds in the 1/18thPC temperament grow consistently wider in both directions away from C up to the peak at E-G#, with no such jumps in size anywhere. Also note that major 3rds on Db and F#, those farthest away from F and C, are of equivalent size in the 1/18thPC temperament (in fact mirroring the matched size of the major 3rds on F and C) lending an interesting kind of harmonic balance to the general scheme. Overall, the 1/18thPC interpretation has both an entirely consistent and more gradual progression in the widening of major 3rds in either direction away from the calmest keys of F and C.

Back to the script itself

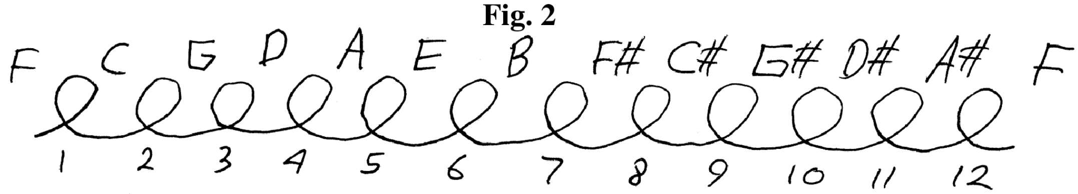
If 18ths were to be the smallest quantity represented in this symbolic scheme, the smallest common denominator, then 1/6th quantities would need to be spelled out, in the language of the script, as -3/18ths. Looking once again at the WTC tuning script *(Figure 1)* it struck me that the single and double loops inside the larger loops were of two legible sizes, even though they didn't need to be of two sizes for aesthetic or calligraphic or any mechanics-of-the-hand reasons I could imagine.

Look closely at the innermost loops of the five triple-loop figures and note their size. Now look closely at the three figures with only a single loop inside of them. None of these three inner loops is larger than any innermost loop of the five initial figures, even though there is ample room to pen larger single loops in these latter three figures. I wondered whether this constant sizing was intentional; whether differently sized inner loops could signify different quantities. Specifically, an easy to interpret ratio of 1 to 2. If so, then any smallest inner loop would equal 1/18th of a comma, and any larger inner loop would equal 2/18ths of a comma. Smaller 1/18th loop + larger 2/18ths loop = 3/18ths! Those first five figures in the script would then be regular,-1/6th comma 5ths, just as in Lehman's 1/12thPC reading. I was intrigued.

That sense of intrigue compelled me to try and understand how someone may have originally come up with this specific representational scheme. It occurred to me that this entire loopish system may have begun as a simple series of zeros used to represent twelve consecutive pure 5ths in a row. If that series of zeros were then penned in one long, connecting stroke, they would form a series of twelve linking loops. Let's refer to these as "5th loops." Additional loops could then, with the same continuous stroke, be easily penned inside some of the 5th loops to show by what degree they were to be altered to become tempered 5ths. Let's call these inner, fractions-of-a-comma-informing-loops "quantifying loops."

Note that I said twelve linking loops, as in the number of single, double and triple loop figures seen in the WTC script *including a twelfth single loop at the very*. Twelve is how many consecutive 5ths you need, in a straight line format, to show a complete distributional spread of the comma. This is because if one represents a series of pure 5ths starting at F, the Pythagorean comma, the amount by which the upper note of the twelfth pure, consecutive 5th overshoots the starting tone, is fully explicated with the inclusion of that final Bb/A# to "F" interval. *(Figure 2)*

Fig. 2



This repeated "F" would actually be somewhat sharp compared to the starting F, by a Pythagorean comma, which is therefore the amount by which some or all of the 5ths must be narrowed (tempered) to make the ending F some true octave to the beginning F. This same series of consecutive 5ths can of course also be represented in a circle without restating the last F, just like on a clock face where "12" begins and ends the circle of hours. That is the typical graphical concept used in so called tuning circles; twelve 5ths comprise the circle, but instead of a "12" at the top of there is the starting tone of the tuning series. So starting with that same F, the A#/Bb would be located at the "11" position. No reason to write another F to close the circle because the starting F stands for both the beginning and the ending of the consecutive 5ths. *(Figure 3)*

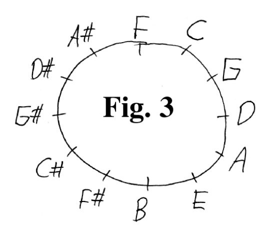


Fig. 3

But it would be natural to restate the beginning F at the end of a linear representation of a temperament. Take another look at the linear tuning "sentences" used on page 2 of this essay. First compare the terms and form of these linear sentences with Figure 2, and then look at the WTC script itself. It's the same kind of linear graphical concept with all twelve 5ths accounted for. Remember these linear versus circular concepts when I come around to discussing that little twelfth loop floating off at the tail of the WTC script.

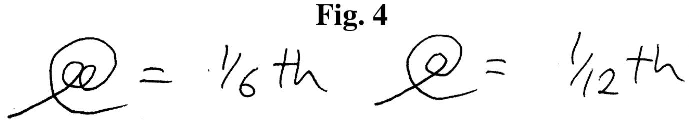
From function to form

Seeing that a 3/18ths temperament was actually quite simple and easy to tune, and having this speculative model for the script's possible origin and structure, the proportionality in the figures comprising the script seemed quite natural. I was then struck by something fairly basic: the temperament was devised and in use before the script was needed or created This turns out to be an important insight because we can then assume that the script's creator conceived it specifically for expressing this temperament, and so would have picked the clearest, least confusing terms and format to achieve clarity in this unique, sentence-like representation.

If the tempered 5ths were to be represented by subtracting one quantity to produce the –1/6th comma 5ths, and some multiple of that quantity to indicate the 1/18th comma 5ths, what would be the easiest, clearest way to convey that information using loops as quantifying terms in a graphical sentence? Three simple concepts occurred to me, followed by a 4th that is problematic for some representations. I think these four possibilities would have occurred to nearly anyone setting out to make this kind of script do what it does with merely a series of loops penned in one stroke.

Concept #1: One could use two quantifying loops of the same size inside any 5th loop to signify twice the quantity of a single such quantifying loop inside any 5th loop, which would not look like the WTC script. *(Figure 4)*

Fig. 4



Concept #2: One could use either of two, easily distinguishable sizes of quantifying loops inside any 5th loop, the smaller size representing a smaller quantity, the larger size a larger quantity, which would look more like the WTC script, but not quite because there would never be two quantifying loops together inside a 5th loop.

Concept #3: One could use two easily distinguishable sizes of quantifying loops to represent a smaller and a larger quantity, appearing either singly or together inside any 5th loop, which would look just exactly like the figures in actual WTC script.

Concept #4: Use two sizes of quantifying loops to represent the same quantity. A small and a large quantifying loop placed together inside a 5th loop equal twice any single loop quantity. This would also look just like the WTC script, but using two sizes of loops to signify one quantity would be inherently, confusingly equivocal. It would lack the clarity of concept #2, and could even be misread as concept #3, where two sizes of quantifying loops easily signify two different quantities.

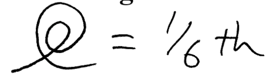
Form and fit for the two interpretations

Lehman decodes the script as per concept #4, such that different sizes of quantifying loops signify the same quantity. Add up all the quantifying loops and you get thirteen such quantities. That's one too many for a series of twelve consecutive 5ths distributing the comma (in no more than two sizes of tempered 5th,) so that last A#/Bb to F 5th must end up being wide by 1/12th PC to make it all come out.

But if the script's creator was trying to clearly represent the B-L1/12thPC temperament, where the fractions to be expressed are either 12ths or 6ths, which concept would be unambiguous <u>and</u> the easy to execute? Because 12ths are half the size of 6ths, concept #1 would have been an acceptable choice because it is easy to execute and to interpret, but it was not employed, possibly because it looks a little spooky, like the ghostly sets of eyes in *Figure 4.*

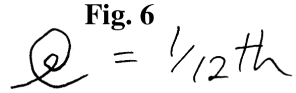
Concept #2 would have been the best choice for the B-L1/12thPC because it would take but one larger loop inside a 5th loop to express the 1/6th quantity; *(Figure 5)*

Fig. 5



or a single, recognizably smaller loop inside a 5th loop to signify 1/12th. *(Figure 6)*

Fig. 6



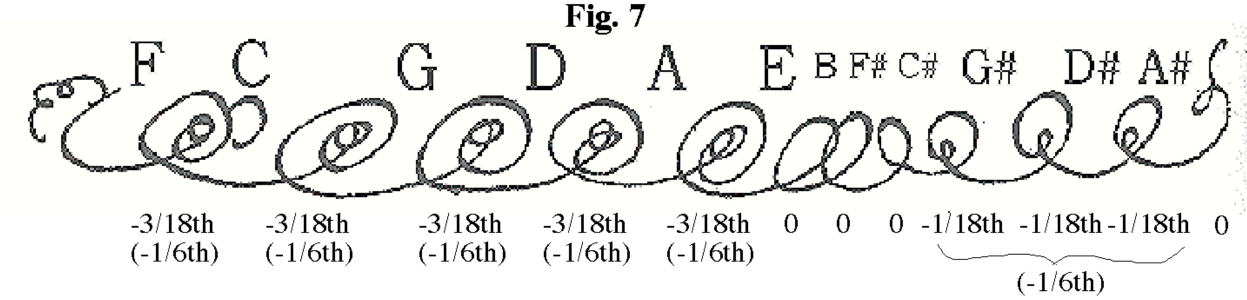
But concept #2 was not employed in the WTC tuning script.
Concept #3 wouldn’t be optimal for the B-L1/12thPC because it naturally seems to express different quantities with different quantifying loop sizes.

Concept #4 would be a poor choice to express the B-L1/12thPC because using paired large and small quantifying loops in some 5th loops, and single, small quantifying loops inside other 5th loops, would inadequately convey the idea of <u>equal quantities</u> from the quantifying loops. It would be an inherently confusing way to express that particular temperament.

But what if this hypothetical 3/18ths temperament were to be represented? Employing concept #1 would have involved the penning of <u>three</u> quantifying loops, worth 1/18th each, inside each of the 1/16th comma 5th loops, which would be over thirty loops to pen in one stroke. Truly scary! Concept #2 would require that the two sizes of inner loops be drawn very, very exactly so that they would show that the larger quantifying loops represented three times (at 3/18ths) the value of the smaller quantifying loops (at 1/18ths.) A pretty practiced eye and hand that would take with a quill pen, and in one stroke!

But concept #3, which would look like the WTC script, is perfect for expressing 3/18ths and 1/18ths quantities, requiring a minimum number of quantifying loops, and using just two, easily distinguishable sizes of those loops. It would be straightforward to read, given a careful look at the entire script, including the upside-down loop at the tail. Looking at the WTW script with these four possible concepts, and the prsent two interpretations in mind, it seemed to me that WTC script was best explained as a representation of the 1/18thPC, 3/18ths solution. What would Holmes think? *(Figure 7)*

Fig. 7



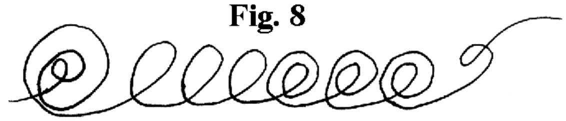
The only slightly debatable language of the script.

After creatively speculating on a possible genesis for the specific language of the script, and seeing how musically “well” it all came out, I believe that it may have been devised to represent a 1/18thPC temperament. Some additional thoughts on the apparent care that went into penning the WTC script may support this assertion:

1. The pairs of quantifying loops inside the beginning set of five tempered 5th loops are consistently legible as being of two sizes, showing a two-to-one relationship.

2. The single quantifying loops inside the ending set of three tempered 5th loops are small, like the smallest quantifying loops inside the beginning set of five 5th loops, even though these quantifying loops in the three ending 5th loops could have been penned larger than we find them: there is plenty of room. They could, in fact, have been easily penned as shown in *Figure 8*.

Fig. 8



Still, I wondered whether there could be any more clues that could clinch the whole thing. Perhaps something that would show not just that the scribe may have had two sizes of quantifying loops in mind, but that he intended and carefully penned two relative sizes of these temperament defining loops. What could show intent here? That the production of two sizes of quantifying loops would not likely have resulted from unconscious or habitual motions of the hand while penning the script?

I looked to see whether the small size of the single quantifying loops in the ending set of three 5th figures may have resulted from a mechanical, repetitive drawing carryover effect. In the initial five tempered 5th figures the smallest quantifying loops were clearly penned first, the larger quantifying loops second, and then the surrounding 5th loops. I assumed that the scribe would also have also penned the single quantifying loops in the ending set of three tempered 5th figures first, as in the initial five tempered 5th figures. But in that ending set of three tempered 5th figures the quantifying loops were in fact penned last, and the outer loops first.

This change in the order of penning would seem to rule out an unconscious, habitual carryover effect from the execution of the initial five figures as the cause of the small sizing of those single quantifying loops in the three ending figures. Could it be that in those three tempered 5th figures the scribe penned each 5th loop before the inner quantifying loop to more easily mimic the shape and size of the preceding pure 5th loops? Whatever the reason for this sequential change in the penning of the smallest quantifying loops, someone carefully chose to maintain their relatively small size.

Temperament by the tail

And what about that single empty loop floating off the very tail of the script? Maybe it has something definitive to add or subtract from this equation, or is it, like a zero, neutral? Floating upside down at the end of the drawn-with-one-stroke script, it's either a nuisance or a blessing depending on what you hope it means. For the 1/18thPC interpretation it can be read as indicating a final pure 5th from A# to F, perhaps penned with the look of an afterthought and upside down because it merely indicates the inevitable disposition of that final, resultant, consecutive 5th, or even to suggest circularity. But given this script's unique sentence-like format that last empty loop may be there to avoid an overall misinterpretation of what is being carefully and completely conveyed.

Moreover, spelling out the disposition of that twelfth 5th could also serve to remind the tuner to check that the closing A# to F does not beat. So, a practical touch. Whatever the case may be, the meaning of that final loop is consistent with the 1/18thPC interpretation. It could also be consistent with the Lehman's /12thPC, drawn upside down to indicate a wide 5th. Could that final upside-down loop be there just for looks? If it was meant to be purely decorative then why make it confusingly similar with figures comprising the language of the script? Since it was drawn as part of the same beginning-to-end stroke it seems odd to me that it wouldn't be an integral part of the sentence.

And what of that separately penned decoration, comprised of two loops, at the other end of the script? Was it meant to be purely decorative? The script would, to my eye, look unbalanced and unadorned without something there, so my guess is that whoever penned the script saw that it looked a bit plain on that end and tacked on a few loops to balance it out. And who knows but that the scribe didn't intend, ever so decoratively, to slightly disguise the fact that there even was a tuning script? That also brings up the question of whether there was some cryptological, secret-keeping purpose to the script. Personally, I think there was no reason to keep a temperament secret at the time, and that this was just an interesting way to express a favored temperament for the WTC right on the manuscript's title page.

Lehman speculates that this two-looped squiggle might indicate the beat rate of the major 3rd from A to middle-C, which is three beats per second for A at 440, and 2.8 at A 415. But why then use loops, already used to represent 5ths and fractions of the comma throughout the script, to convey the unrelated concept of beat rates? And does the doodling at the bottom of the page have meaning too? (I must confess that I have some faint thoughts on both these loopy items, but they are undeveloped so I'll spare you their expression until such time that they coalesce into something firmer, if they ever do.)

Occam's Razor

Recalling the observations and points made in this little inquiry, the simplest, most consistent, most complete interpretation of the WTC tuning script may be this 18ths solution. Even so, barring some definitive, external corroboration, I believe this and all other readings of the riddle should always be presented as hypothetical <u>interpretations</u> of the possible WTC tuning script. The whole thing could be a randomly penned decoration that amazingly but foolishly can be seen as a tuning recipe! I look forward to reading other interpretations as they come along, which they surely will!

Please keep in mind that my reading of the script conceptually piggy-backs on Andreas Sparschuh's original idea about the 5ths-figures language of WTC tuning script, and on Bradley Lehman's idea that fractions of a comma were in fact being depicted by the inner loops. And I truly appreciate the many useful and patient comments from Bradley and other members of the incredibly informative Harpsichord and Related Topics List (http://www.albany.edu/faculty/bec/hpschd-l/).

Charts and tuning.
I recommend Lehman's method for setting up the series of regular (-1/6th comma) 5ths. It can be found at: http://www-personal.umich.edu/~bpl/larips/tetrasect.html I provide a beat rate chart (http://bachtuning.jencka.com/charts.pdf) to help one more easily see the resulting beat-speed relationships of 5ths, 4ths, and major and minor 3rds, and to spot proportionally beating interval sets that could be useful in tuning.

For example, after tuning the five 1/6th comma narrowed 5ths that determine F, C, G, D, A and E, one can check to see that the e–a 4th (below middle-c) beats at precisely half the speed of the f–a major 3rd. If the e–a 4th beats at less than half the speed of the f–a major 3rd it means you have made one or several -1/6th comma 5ths too wide. Conversely, if the e–a 4th beats at more than half the speed of the f–a major 3rd then one or several -1/6th comma 5ths are too narrow. Also, c–e should beat three times for every two beats of e–a. And after tuning pure 5ths from E to B to F# to C# (and from Bb to F in the 1/18th PC,) one sets the three -1/18th 5ths from C# to G# to D# to (the already tuned Bb) as just barely beating. The d#–g major 3rd should then have the same speed as the g–b major 3rd above it.

Enjoy!

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